INFLUENCE OF CONTACTS ON THE THERMAL RESISTANCE

OF A BUNDLE OF RECTANGULAR FINS

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Recommendations are given for the calculation of the heat-transfer coefficient through the base of a bundle of rectangular fins, taking account of the contact resistance between its elements.

In the practical development of miniature electrical devices, for example, semiconductor heat pumps, compact plate heat-exchangers are used, in contact with the heat source through their base. The complexity of producing such a heat exchanger in the form of a monolithic structure justifies the spread of "composite" construction, in the form of a bundle of alternating metallic rectangular fins and intervening distance pieces; the latter, together with the parts of the fins adjacent to the base, serve the function of supporting walls, which are rendered artificially monolithic by means of glue or mechanical combination of its elements.

In calculating the heat-transfer resistance through the base of such heat exchangers (Fig. 1), in contrast to traditional units in which the fin and the wall form a continuous structure [1], it is necessary to take account not only of convective heat transfer at the open surface of the fins but also of heat transfer in the contact region with the distance pieces.

Assuming that α_0, t_0 are constant over the height of the fin, that the distance-piece thickness δ is equal to the width of the interfin space, that r_c is constant over the height h of the distance piece, and also that (assumptions which are correct for compact heat exchangers) $\delta/2r_c\lambda_d \ll 1$, $\Delta\alpha_0/2\lambda_f \ll 1$ (in practice, <0.1 [2]), $l \gg \delta$, $l \gg \Delta$, the calculation reduces to integrating a system of differential equations of one-dimensional heat conduction for the fins and distance pieces in section 1 $(0 \leqslant X_1 \leqslant h)$:

$$d^2t_{f1}/dX_1^2 = m_{f1}^2(t_{f1} - t_d), \tag{1}$$

$$d^2t_d/dX_1^2 = m_d^2(t_d - t_{f1}), \tag{2}$$

and also integrating the heat-conduction equation for the fin in section 2 $(0 \leqslant X_2 \leqslant H)$ with convective heat transfer:

$$d^2t_{f2}/dX_2^2 = m^2(t_{f2} - t_0). \tag{3}$$

As a result, expressions are obtained for the longitudinal temperature distribution in the fins and distance pieces:

$$t_{f1} = C_1 \exp MX_1 + C_2 \exp(-MX_1) + C_3X_1 + C_4, \tag{4}$$

$$t_d = -\omega[C_1 \exp MX_1 + C_2 \exp(-MX_1)] + C_3X_1 + C_4, \tag{5}$$

$$t_{f2} = C_5 \exp mX_2 + C_6 \exp(-mX_2) + t_0. \tag{6}$$

The constants C_1–C_6 of the integration are found from the boundary conditions

$$X_1 = 0, \quad t_{f1} = t_a + r_{c.f}\lambda_f \frac{dt_f}{dX_1};$$

$$X_1 = 0, \quad t_d = t_a + r_{c.d}\lambda_d \frac{dt_d}{dX_1};$$

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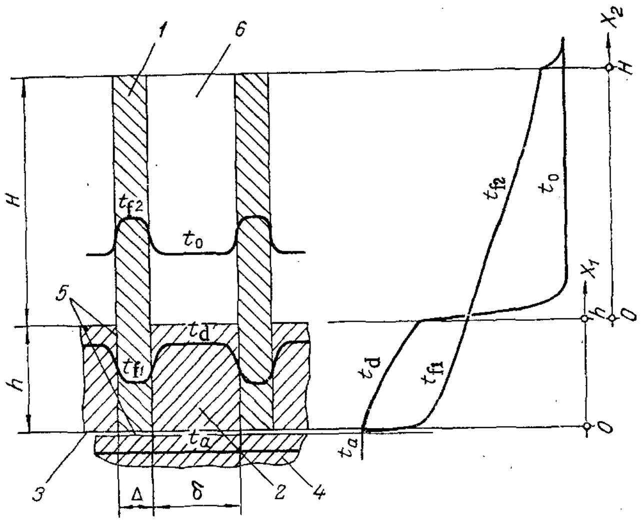

Fig. 1. Calculation scheme: 1) fin; 2) distance piece; 3) base of bundle; 4) isothermal wall; 5) contact zone; 6) heat carrier.

$$X_1 = h, \quad \alpha_0 (t_d - t_0) = -\lambda_d \frac{dt_d}{dX_1};$$

$$X_1 = h, \quad X_2 = 0, \quad t_{f1} = t_{f2};$$

$$X_1 = h, \quad X_2 = 0, \quad \frac{dt_{f1}}{dX_1} = \frac{dt_{f2}}{dX_2};$$

$$X_2 = H, \quad \alpha_T (t_{f2} - t_0) = -\lambda_f \frac{dt_{f2}}{dX_2}.$$

According to the heat balance in the plane $X_1 = 0$, using Eqs. (4) and (5), the heat-transfer coefficient through the base of the bundle of fins and distance pieces is determined from the relation

$$k = -\frac{\lambda_f \dfrac{\Delta}{\Delta + \delta} \left.\dfrac{dt_{f1}}{dX_1}\right|_{X_1=0} + \lambda_d \dfrac{\delta}{\Delta + \delta} \left.\dfrac{dt_d}{dX_1}\right|_{X_1=0}}{t_a - t_0} = -\lambda_{ef} \frac{C_3}{t_a - t_0};$$

after determining C_3, an expression is obtained for the effective dimensionless resistivity to heat transfer

$$\beta_{ef} = \eta + \beta_{c.d} L + \frac{C + D\psi + E\sqrt{1-\psi^2}}{A + B\psi}, \tag{7}$$

where

$$A = \frac{\rho^2}{\mu} \sqrt{2/\beta_c L}\, [\mu (\beta_0 \mu L + \Phi \sqrt{\beta_0 L/2}) + \beta_{c.f} + \beta_{c.d} \mu^2 L];$$

$$B = \rho^3 + 2 (\beta_{c.f} + \beta_{c.d} \mu^2 L) (\beta_0 L \mu + \Phi \sqrt{\beta_0 L/2}) \frac{\rho}{\beta_c \mu L};$$

$$C = \frac{\sqrt{2/\beta_c L}}{\mu} \{\Phi \mu \sqrt{\beta_0 L/2}\, [\mu^2 (2\beta_{c.f} - \beta_{c.d} L + \beta_{c.d} L \mu^2) + \beta_0 \rho^4 L] + \beta_0 L (\beta_{c.f} + \beta_{c.d} \mu^2 L) + \mu^2 L (\beta_{c.f} - \beta_{c.d} L) (\beta_0 \mu^2 + \beta_{c.d} \rho^2)\};$$

$$D = \beta_0 \rho L + \mu^2 \rho (\beta_{c.f} - \beta_{c.d} L) (1 + 2\beta_0 L \beta_{c.d}/\beta_c) + \left[2\mu\rho \frac{\beta_{c.d}}{\beta_c} (\beta_{c.f} - \beta_{c.d} L) + \mu^3 \rho + \frac{2\rho^3 \beta_0}{\mu \beta_c} (\beta_{c.f} + L \mu^2 \beta_{c.d})\right] \Phi \sqrt{\beta_0 L/2};$$

$$E = 2\mu\sqrt{2/\beta_c L}\ (\beta_{c.f} - \beta_{c.d} L)(\beta_0 L - \Phi\mu\sqrt{\beta_0 L/2});$$

$$\psi = \operatorname{th}\eta\frac{\rho}{\mu}\sqrt{2/\beta_c L};$$

$$\Phi = \frac{(\beta_T\sqrt{2/\beta_0 L} + \mu)(\beta_T\sqrt{2/\beta_0 L} - \mu)^{-1}\exp 2\frac{\theta}{\mu}\sqrt{2/\beta_0 L} + 1}{(\beta_T\sqrt{2/\beta_0 L} + \mu)(\beta_T\sqrt{2/\beta_0 L} - \mu)^{-1}\exp 2\frac{\theta}{\mu}\sqrt{2/\beta_0 L} - 1};$$

when $\alpha_T = 0$ (heat-insulated ends of the fins)

$$\Phi = \operatorname{cth}\frac{\theta}{\mu}\sqrt{2/\beta_0 L}.$$

Analysis of Eq. (7) leads to the following observations.

1. With the obvious tendency toward decrease in heat-transfer coefficient k with increase in the contact thermal resistivities r_c, $r_{c.f}$, $r_{c.d}$, the latter influence k in different ways

$$\lim_{r_{c.f},\, r_{c.d}\to\infty} k = 0, \quad \lim_{r_c\to\infty} k > 0;$$

with increase in $r_{c.f}$, $r_{c.d}$, the influence of r_c on k is reduced; there may arise a situation in which k is independent of r_c, specifically, with simultaneous satisfaction of the conditions

$$\vartheta = 0, \quad x = 0;$$

this occurs practically when $\alpha_T = 0$, for example,

$$r_{c.f}\lambda_f = r_{c.d}\lambda_d, \quad \sqrt{2\beta_0 L}/\mu \gg 1, \quad H = 0.5\Delta\lambda_f/\lambda_d.$$

2. The height h of the lateral-contact zone (the distance-piece height) influences k in a complex manner. Thus, when

$$\beta_c < \frac{2}{L\rho^2}\frac{(ux - \mu\vartheta z)^2}{(\mu z + u)^2 + \mu^2(\vartheta + x)^2} \tag{8}$$

there is an optimal value of h which corresponds to a maximum of k. The optimum is explained by the improvement in heat transfer with increase in lateral-contact surface and simultaneously its deterioration with accompanying increase in thermal resistance of the distance piece. Equation (8) does not hold with sufficiently small (but with satisfaction of the condition $\delta/2r_c\lambda_d < 0.1$ of the one-dimensional scheme) values of r_c; k decreases here with increase in h and, in the particular case when the conditions $\vartheta = 0$, $x = 0$ are satisfied, this decrease occurs according to the law $k \sim h^{-1}$; the analytical expression for h_{op}, the optimal height of the distance piece, may be found in the particular case encountered in the practical construction of efficient heat exchangers, i.e., when $\alpha_T = 0$, $L = 1$, $\beta_{c.d} = \beta_{c.f} = \beta_c$; here $E = 0$ and

$$\eta_{op} = \frac{h_{op}}{\delta} = \frac{\mu}{2\rho}\sqrt{\beta_c/2}\ \ln\frac{1+\psi_{op}}{1-\psi_{op}}, \tag{9}$$

where

$$\psi_{op} = \frac{AB}{K+B^2}\left[\sqrt{1 + \frac{(K-A^2)(K+B^2)}{(AB)^2}} - 1\right];$$

$$K = \frac{\rho}{\mu}\sqrt{2/\beta_c}\,(BC - AD).$$

3. There always exists an optimal value of the fin thickness Δ (at which k is a maximum). The relation of Δ to r_c, $r_{c.d}$, $r_{c.f}$, h and also other geometric characteristics is found by numerical methods, in view of its complexity.

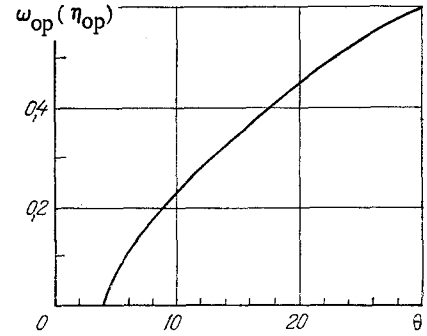

Fig. 2. Dependence of ω_{op} on θ at optimal η in the range β_c = 20-200.

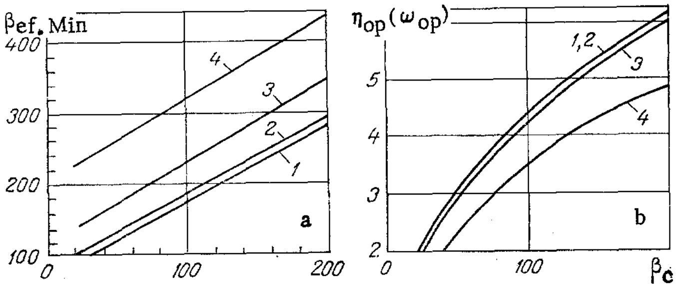

Fig. 3. For the calculation of the maximum heat-transfer coefficient (a) and correspondingly the optimal height of the distance piece (b) as a function of β_c and θ at optimal fin thickness: 1) θ = 30; 2) 20; 3) 10; 4) 5.

In developing compact efficient heat exchangers, optimizing the construction reduces, as a rule, to determining the geometric parameters ensuring k_{max} with a series of specified values, for example, δ, H. Here, on the basis of calculation by the above-outlined method, the quantities h_{op} and Δ_{op} (or the dimensionless equivalents η_{op} and ω_{op}) must be determined, and then Δ is decreased and h increased to economize the fin mass and increase the strength of the structure without perceptible reduction in k with respect to its maximum.

As a characteristic example of the solution of this problem, consider the case when α_0 depends on the fin height H; this case is encountered, in particular, in laminar stabilized motion of the heat carrier in rectangular channels. Here the Nusselt number Nu = $\alpha_0 d_e/\lambda_0$ may be written in the form

$$\mathrm{Nu} = 3.26 + 4.622\,(1 - \theta)^{3.33} \tag{10}$$

where $d_e = 2\delta(1 + \theta^{-1})^{-1}$.

It is assumed that the conditions under which Eq. (9) is obtained hold: specifically, that the ends of the fins are heat insulated, that the fins and distance pieces are made from the same material ($\lambda_d = \lambda_f = \lambda$), and that the contact thermal resistivities are the same ($r_{c.d} = r_{c.f} = r_c$). Then, according to Eq. (10), β_0 and correspondingly β_{ef} are functions of the ratio λ/λ_0. It is assumed that λ/λ_0 = 6000, which is characteristic for heat transfer of a flow of atmospheric air with heat-exchanger elements made from aluminum alloy. The results of analyzing the dependences $\eta_{op}(\omega, \beta_c, \theta)$, $\beta_{ef.min}(\omega, \beta_c, \theta)$, and $\beta_{ef}(\omega, \beta_c, \theta, \eta)$ when $\eta < \eta_{op}$ and $\eta > \eta_{op}$ according to Eqs. (9) and (7) lead to the following conclusions:

the optimal dimensionless height η_{op} of the distance piece at first rises with increase in the dimensionless fin thickness ω and then decreases; in the ranges θ = 5-30 and β_c = 20-200, the maximum value of η_{op} is practically independent of the fin height H and is related to β_c as follows:

$$\eta_{op\,max} \simeq 0.429\sqrt{\beta_c};$$

the corresponding critical value of ω is found in the range ω_{cr} = 0.3-0.4; it is expedient to ensure that $\eta > \eta_{op}$, since the optimal height of the distance piece is insufficiently large (especially at small β_c) to ensure the required strength of the structure;

with increase in ω, the minimal dimensionless resistivity to heat transfer (calculated at η_{op}) $\beta_{ef.min}$ at first decreases and then increases; the optimal (corresponding to a minimum of $\beta_{ef.min}$, i.e., the quantity $\beta_{ef.Min}$) ω is practically independent of β_c and increases markedly with increase in θ (Fig. 2);

the smallest of the minimal $\beta_{ef} \equiv \beta_{ef.Min}$ (found at optimal η and ω) increases linearly with increase in β_c and nonlinearly with decrease in θ; $\theta = 30$ is practically the limit: $\beta_{ef.Min}$ reaches its smallest value (Fig. 3a); at optimal ω, η_{op} increases nonlinearly with increase in β_c and θ (Fig. 3b);

with significant deviations of ω "to the left" and "to the right" of ω_{op} and of η "to the right" of η_{op}, β_{ef} passes through a very weak minimum: ω_{op} and η_{op} are not clearly expressed; for example, when $\eta = \eta_{op}$ and $\theta = 30$, $\omega_1 = \omega_{op}(\eta_{op})/3$ and $\omega_2 = 3\omega_{op}(\eta_{op})$, it is found that: for $\beta_c = 20$, $\beta_{1ef} = 1.15\beta_{ef.Min}$, $\beta_{2ef} = 1.27\beta_{ef.Min}$: for $\beta_c = 200$, $\beta_{1ef} \simeq \beta_{2ef} = 1.06\beta_{ef.Min}$: when $\eta = 10\eta_{op}$ and $\theta = 30$, $\omega = \omega_{op}(\eta_{op})/3$, $\beta_c = 20$-200, it is found that $\beta_{ef} \simeq 1.25\beta_{ef.Min}$;

with limiting values $\omega = \omega_{li}$, $\eta_{op} = 0$ is ensured; when $\omega \geqslant \omega_{li}$, $\beta_{ef}(\eta)$ varies from $\beta_{ef.min}$ to ∞ with increase in η from 0 to ∞. The dependence $\omega_{li} = f(\theta, \beta_c)$ is shown in Fig.4.

Thus, the choice of an optimal construction of efficient heat exchangers with specified δ, H, r_c may reduce to determining the pair of optimal values of ω (Fig. 2) and η (Fig. 3b). To decrease the heat-exchanger mass and its resistance to heat-carrier motion, a 2-3-fold reduction in fin thickness Δ is permissible and to increase the strength of the joints between elements, a 2-10-fold increase in distance-piece height h in comparison with the optimal value; the heat-transfer coefficient may remain very close to its maximum here. For example, in the case of laminar stabilized heat-carrier motion in plane-slit channels, when $r_c = r_{c.f} = r_{c.d} = 4\cdot10^{-4}$ $W^{-1}\cdot m^2\cdot K$, $\delta = 10^{-3}$ m, $H = 30\cdot10^{-3}$ m, $\lambda_0 = 0.025$ $W\cdot m^{-1}\cdot K^{-1}$, $\lambda_f = \lambda_d = \lambda = 150$ $W\cdot m^{-1}\cdot K^{-1}$, $\alpha_T = 0$, it is found that: the greatest value of $k = 1110$ $W\cdot m^{-2}\cdot K^{-1}$ is ensured at optimal $\Delta = 0.6\cdot10^{-3}$ m and $h = 3.4\cdot10^{-3}$ m; with decrease in Δ to $0.25\cdot10^{-3}$, k decreases by 12.5%, the heat-exchanger mass decreases by 49%, and the area of the active cross section increases by 28% (which significantly reduces the loss in heat-carrier head); further, with doubling in the distance-piece height h, which may be sufficient to ensure the necessary strength of the supporting wall, k decreases altogether by 1%; the resulting reduction in heat-exchanger mass here is 41%; even with a tenfold increase in h with respect to the optimal value, there is a resulting reduction in heat-exchanger mass (13%) thanks to decrease in Δ; the reduction in k here is 24.4%.

NOTATION

X, current coordinate; h, H, l, distance-piece height, height of fin with open surface, fin length; Δ, δ, thickness of fin and distance piece; t_α, t_{f_1}, t_{f_2}, t_d, t_0, temperatures of isothermal wall, fin in section 1, fin in section 2, distance piece, and heat carrier; r_c, $r_{c.f}$, $r_{c.d}$, contact thermal resistivities of side surfaces of fins with distance pieces, base fin with isothermal wall, distance piece with isothermal wall; α_0; α_T, mean heat-transfer co-

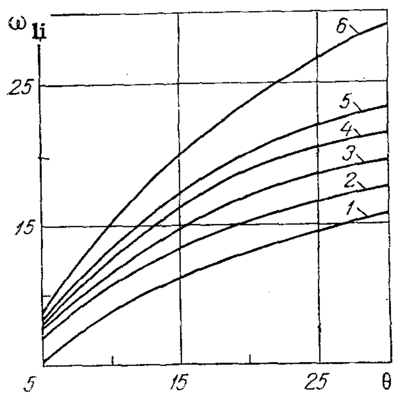

Fig. 4. Limiting values of ω corresponding to $\eta_{op} = 0$ as a function of θ and β_c: 1) $\beta_c = 20$; 2) 40; 3) 60; 4) 80; 5) 100; 6) 200.

efficients at the open side surfaces of the fins and the ends of the distance pieces, at the ends of the fins; λ_o, λ_f, λ_d, thermal conductivities of the heat carrier, fin material, and distance-piece material;

$$m^2 = 2\alpha_0/\Delta\lambda_f;\ m_{f1}^2 = 2/r_c\Delta\lambda_f;\ m_d^2 = 2/r_c\delta\lambda_d;\ \omega = \Delta/\delta L;\ L = \lambda_d/\lambda_f;\ \mu = \sqrt{\omega};\ \rho = \sqrt{1+\omega};\ M = \sqrt{m_{f1}^2 + m_d^2};$$

$$\lambda_{ef} = (\Delta\lambda_f + \delta\lambda_d)/(\Delta + \delta);\ \beta_{ef} = \lambda_{ef}/\delta k;\ \beta_{c.f} = r_{c.f}\lambda_f/\delta;\ \beta_{c.d} = r_{c.d}\lambda_f/\delta;\ \beta_T = \lambda_f/\delta\alpha_T;$$

$$\beta_0 = \lambda_f/\delta\alpha_0;\ \beta_c = r_c\lambda_f/\delta;\ \theta = H/\delta;\ \eta = h/\delta;\ u = \beta_{c.f} + L\mu^2\beta_{c.d};\ \vartheta = \beta_{c.f} - L\beta_{c.d};\ x = \Phi\mu\sqrt{\beta_0 L/2} - \beta_0 L;\ z = \Phi\sqrt{\beta_0 L/2} + \beta_0 L\mu.$$

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DYNAMIC VISCOSITIES OF MIXTURES OF n-BUTYRALDEHYDE AND ISOBUTYRALDEHYDE OVER WIDE RANGES IN THE STATE PARAMETERS

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Dynamic-viscosity measurements are reported for liquid mixtures in the system formed by n-butyraldehyde and isobutyraldehyde.

The dynamic viscosity has been measured with an apparatus based on the capillary method involving the use of a viscometer designed by Golubev [1]. The viscosity has been measured over the temperature range 293-503°K and the pressure range 0.1-58.9 MPa. The basic viscometer parameters at room temperature were as follows: capillary radius 1.1×10^{-4} m and length 9.05×10^{-4} m, volume of measurement vessel 214.13×10^{-4} m³. The geometrical dimensions of the viscometer were determined with an MIR microscope and KM-8 cathetometer by the method of [2].

TABLE 1. Dynamic Viscosities of Mixtures of n-Butyraldehyde and Isobutyraldehyde, $\eta \times 10^7$ Pa·sec

T, K	P, MPa							
	0,1	5,0	9,9	19,7	29,5	39,3	49,1	58,9
88% n-butyraldehyde+12% isobutyraldehyde								
293,51	23207,0	24200,0	25209,0	27211,5	29213,8	31216,0	33218,3	35220,5
317,08	17252,3	17998,2	18744,2	20236,2	21728	23220,1	24712,0	26204,0
333,89	14344,2	14985,7	15627,4	16910,9	18194,2	19477,5	20761,0	22044,2
364,90	—	11172,5	11702,0	12761,0	13820,0	14879,0	15938,0	16997,0
399,90	—	8407,4	8874,5	9808,8	10743,0	11677,3	12611,5	13545,8
434,52	—	6429,0	6866,3	7740,9	8615,6	9490,2	10364,8	11239,4
465,02	—	5159,2	5587,1	6443,1	7299,0	8155,0	9010,9	9866,8
483,05	—	4548,9	4976,2	5830,9	6685,6	7540,3	8395,0	9250,0
503,51	—	3963,0	4393,1	5153,5	6114,0	6974,4	7835,0	8695,3
80% n-butyraldehyde and 20% isobutyraldehyde								
293,50	23069,3	24074,4	25079,7	27090,4	29101,0	31111,5	33122,1	35132,6
318,11	16984,7	17725,2	18465,8	19946,8	21427,8	22908,8	24389,9	25870,9
333,86	14271,3	14915,0	15559,0	16846,5	18134,2	19421,8	20709,4	21997,0
364,15	—	11182,0	11714,7	12780,0	13845,4	14910,8	15976,2	17041,5
399,10	—	8368,0	8835,3	9769,7	10704,0	11638,4	12572,7	13507,1
424,60	—	6863,6	7306,0	8190,4	9075,0	9960,0	10844,0	11728,5
454,96	—	5476,6	5905,1	6762,1	7619,0	8476,0	9333,0	10190,0
473,12	—	4795,7	5222,0	6074,7	6927,3	7780,0	8632,6	9485,3
503,00	—	3876,0	4306,5	5167,5	6028,6	6890,0	7750,6	8611,7

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